Adverse Selection

Asymmetric information

- We analyze the situation where the agent has some information that the principal does not have about some aspect of product/service quality.
- The basic problem is the market for lemons (Akerlof (1970)) where markets collapse.
- The problem can be solved if the informed agent moves first and signals his type (Spence (1973)).
- The problem can be solved if the principal designs a contract to screen the agents (Mirrlees (1971)).

The Market for Lemons

- The market for lemons shows that markets can collapse because of asymmetric information.
- *Lemons* are used cars of low quality, whereas *peaches* are used cars of good quality.
- In the market for second-hand cars, the seller knows the quality of the car but not the buyer.
- Suppose that quality is $k \in [0, 1]$ uniformly distributed.

The Market for Lemons (Cont.)

- Cars of quality 0 are the worst and cars of quality 1 are the best.
- Sellers of good cars have a reservation price. A seller is willing to sell a car of quality k at $p_0 \cdot k$.
- Buyers value the car at $p_1 \cdot k$ with $p_1 > p_0$.
- Assume $p_1 = \frac{3}{2} p_0$.

Equilibrium for the Market

- If information was symmetric, a car of quality k would be sold at a price p between $p_0 \cdot k$ and $p_1 \cdot k$.
- Given that the quality is not observed, there is a unique price p at which all cars are sold.
- The buyer knows that the only sellers who would be willing to sell their car at p are sellers for whom $p_0 \cdot k \leq p$ or $k \leq \frac{p}{p_0}$. Thus, the expected quality of the car is $E(k) = \frac{1}{2} \cdot \frac{p}{p_0}$ and the expected valuation of the buyer is (recall $p_1 = \frac{3}{2}p_0$)
$$p_1 E(k) = \frac{3}{4}p.$$
- Naturally, *no buyer is willing to spend p to buy a car of value $\frac{3}{4}p$, and no car is sold* in the market.

How Can One Solve the Market for Lemons?

- We need to find a way to distinguish between good and bad cars.
- We can utilize *certification* where a third party is trusted to certify cars.
- We can offer a *warranty* where only sellers of good cars are willing to offer the warranty; that is, the warranty acts as a *signal* that the car is good.
- *Lemon laws* were enacted five years after the publication of this paper. These state laws provide remedies to consumers for automobiles that repeatedly fail to meet certain standards of quality and performance.

Examples of Signaling

- Product warranties signal quality.
- An expensive art work in the lobby of a bank signals financial security.
- A dividend policy signals value of the firm to the shareholders.
- Price signals quality of wine.
- Education signals quality of performance.

Education as a Signal

- Agents are born with a productivity level θ, where $\theta = \theta^H$ with probability p and $\theta = \theta^L$ with probability $1 - p$.
- Agents select their level of education, $e \in \{0, 1\}$.
- High productivity agents suffer a cost c^H for acquiring one unit of education, whereas low productivity agents suffer a cost c^L for acquiring one unit of education.
- We assume that $c^H < c^L$.

The Different Types of Equilibria

- A separating equilibrium where high types choose $e = 1$ and low types choose $e = 0$ (S1).
- A pooling equilibrium where both types choose $e = 0$ (P1).
- A pooling equilibrium where both types choose $e = 1$ (P2).

The Beliefs of Employers

- In a separating equilibrium, **agents getting education result in the employer offering them a wage $w^H = \theta^H$; otherwise, $w^L = \theta^L$.**

- In a pooling equilibrium, along the equilibrium, the employers cannot distinguish between low and high types: the employer will pay a wage corresponding to the expected productivity $p\theta^H + (1-p)\theta^L$.

The Separating Equilibrium S1

- We write down the two *incentive* conditions for the H and L agents.
- H prefers to choose $e = 1$ if
$$\theta^H - c^H \geq \theta^L. \ \ [1]$$
- L prefers to choose $e = 0$ if
$$\theta^L \geq \theta^H - c^L. \ \ [2]$$
- This equilibrium exists if
$$c^L \geq \Delta\theta \geq c^H.$$

Note: $\theta^H - \theta^L \geq c^H \ \ [1] \ \ \& \ \ c^L \geq \theta^H - \theta^L \ \ [2]$ where $\Delta\theta = \theta^H - \theta^L$.

The Pooling Equilibrium P1

- Let π be the probability that an agent choosing $e = 1$ is of high productivity. The LHS is the expected productivity.
- Agent H prefers to choose $e = 0$ if
$$p\theta^H + (1-p)\theta^L \geq \pi\theta^H + (1-\pi)\theta^L - c^H.$$
- Agent L prefers to choose $e = 0$ if
$$p\theta^H + (1-p)\theta^L \geq \pi\theta^H + (1-\pi)\theta^L - c^L.$$
- The only binding constraint is the constraint for the H type.
- This equilibrium exists if
$$c^H \geq (\pi - p)\Delta\theta$$
where $\Delta\theta = \theta^H - \theta^L$.

The Pooling Equilibrium P2

- Let π be the probability that an agent choosing $e = 0$ is of high productivity.
- Agent H prefers to choose $e = 1$ if
$$p\theta^H + (1-p)\theta^L - c^H \geq \pi\theta^H + (1-\pi)\theta^L.$$
- Agent L prefers to choose $e = 1$ if
$$p\theta^H + (1-p)\theta^L - c^L \geq \pi\theta^H + (1-\pi)\theta^L.$$
- The only binding constraint is the constraint for the L type.
- This equilibrium exists if
$$c^L \leq (p-\pi)\Delta\theta$$
where $\Delta\theta = \theta^H - \theta^L$.

Education as a Signal

- We have found that there are three equilibria.
- One separating equilibrium, where the high productivity agents get education but not the low ones.
- Two pooling equilibria, where either no type gets education or both types get education.

Screening and Adverse Selection

- Suppose now that the sequence of moves is reversed.
- The uninformed agent (P) (mneumonic for *Principal*) moves first, and the informed agent (A) moves second.
- The principal proposes a *screening contract* such that low types and high types choose different alternatives.
- We will study this optimal screening contract in the context of a relation between an employer and an employee.

Types of Agents

- There are two types of employees: the good ones that have a cost of effort $v(e)$, and the bad ones that have a cost of effort $kv(e)$ where $k > 1$.
- The proportion of good types is p whereas the proportion of bad types is $1 - p$.
- $\Pi(e) - w$ is the expected output minus wage; that is, the principal's profit.
- $\underline{U}$ is the reservation utility of the agent whose utility function is given by $u(w)$.

Optimal Contracts with Symmetric Information

- P chooses to maximize $\Pi(e) - w$ subject to the participation constraint $u(w) - v(e) \geq \underline{U}$.

- The contract for the G type is

$$\begin{aligned} u(w^G) - v(e^G) &= \underline{U}, \\ \Pi'(e^G) &= \frac{v'(e^G)}{u'(w^G)}. \end{aligned}$$

- The contract for the B type is

$$\begin{aligned} u(w^B) - kv(e^B) &= \underline{U}, \\ \Pi'(e^B) &= \frac{kv'(e^B)}{u'(w^B)}. \end{aligned}$$

Optimal Contracts with Symmetric Information (Cont.)

- We compute the Lagrangian for the good type

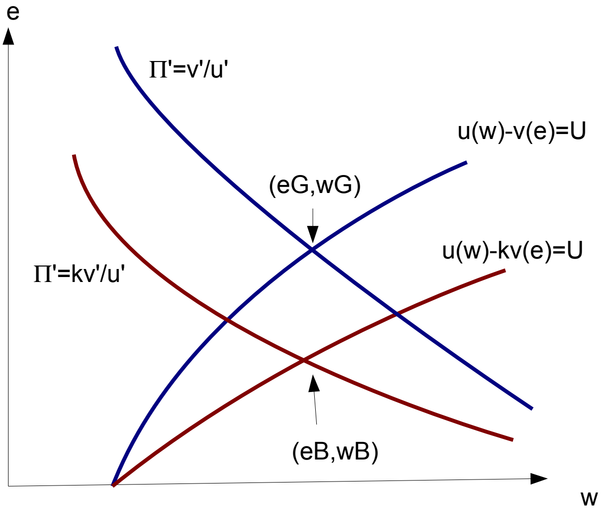
$$\begin{aligned}\mathcal{L} &= \Pi(e^G) - w^G \\ &- \lambda(u(w^G) - v(e^G) - \underline{U}).\end{aligned}$$

- Taking the first order conditions, we get:

$$\begin{aligned}\Pi'(e^G) + \lambda v'(e^G) &= 0 \quad or \quad \Pi'(e^G) = -\lambda v'(e^G) \quad [1] \\ -1 - \lambda u'(w^G) &= 0 \quad or \quad 1 = -\lambda u'(w^G). \quad [2]\end{aligned}$$

- Dividing the two equations gives you the result of the previous slide.

Optimal Contract



Optimal Contract with Asymmetric Information

- If P does not know the type of the agent, offering the two optimal contracts (w^B, e^B) and (w^G, e^G) will not work.
  - The B type will choose (w^B, e^B).
  - The G type will also choose (w^B, e^B) which gives them higher utility than (w^G, e^G).
- The optimal contract under asymmetric information must take this into account and introduce an *incentive constraint*, stating that each type prefers the contract that is directed to that type.

The Principal's Problem

- The principal maximizes $p(\Pi(e^G) - w^G) + (1-p)(\Pi(e^B) - w^B)$ subject to
  - $u(w^G) - v(e^G) \geq \underline{U}$ (PG);
  - $u(w^B) - kv(e^B) \geq \underline{U}$ **(PB)**;
  - $u(w^G) - v(e^G) \geq u(w^B) - v(e^B)$ **(IG)**;
  - $u(w^B) - kv(e^B) \geq u(w^G) - kv(e^G)$ (IB).

The Participation Constraint (PG) Is Not Binding

$$
\begin{aligned}
u(w^G) - v(e^G) &\geq u(w^B) - v(e^B) \text{ by IG} \\
&> u(w^B) - kv(e^B) \textbf{ because } \mathsf{k} > 1 \\
&\geq \underline{U} \text{ by PB.}
\end{aligned}
$$

- This shows that the participation constraint of the good type is never binding and can be ignored.

The Incentive Constraint (IB) Is Not Binding

- Suppose to the contrary that IB is binding. Because $k > 1$, IG cannot be binding.
- Hence $u(w^G) - v(e^G) > u(w^B) - v(e^B)$.
- Because IB is binding, $u(w^G) - v(e^G) > u(w^G) - kv(e^G) = \underline{U}$.
- Suppose that P lowers w^G by ϵ. The profit of the principal increases and the constraints IB,IG and PG remain valid.
- Thus, it cannot be optimal to have IB binding.

Binding Constraints

- In the optimal contract, PB and IG are binding; that is,

$$\begin{aligned} u(w^B) - kv(e^B) &= \underline{U} \\ u(w^G) - v(e^G) &= u(w^B) - v(e^B) > \underline{U}. \end{aligned}$$

- The bad type is at the reservation utility level.
- The good type gets a utility above the reservation utility; that is, an **informational rent**.

Optimal Contract

- We compute the Lagrangian

$$\begin{aligned}\mathcal{L} &= p(\Pi(e^G) - w^G) + (1-p)(\Pi(e^B) - w^B) \\ &+ \lambda(\underline{U} - u(w^B) + kv(e^B)) \\ &+ \mu(u(w^G) - v(e^G) - u(w^B) + v(e^B)).\end{aligned}$$

- Taking the first order conditions, we get:

$$\begin{aligned} p\Pi'(e^G) - \mu v'(e^G) &= 0, \\ -p + \mu u'(w^G) &= 0, \\ (1-p)\Pi'(e^B) + k\lambda v'(e^B) + \mu v'(e^B) &= 0, \\ -(1-p) - \lambda u'(w^B) - \mu u'(w^B) &= 0. \end{aligned}$$

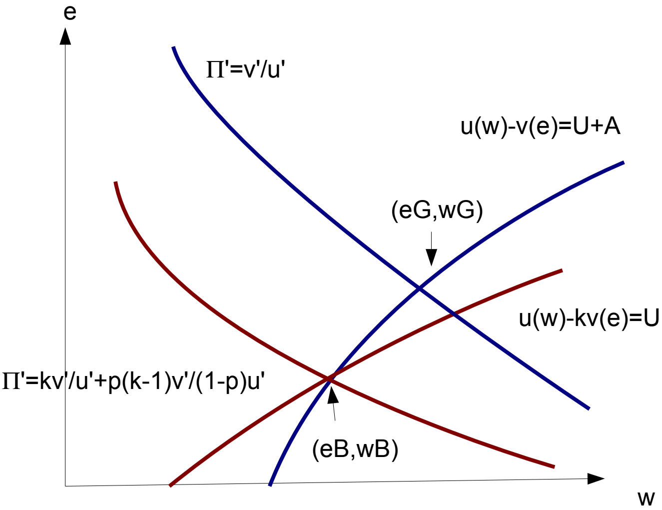
Optimal Contract (Cont.)

- Solving the first order conditions, we obtain:

$$\begin{aligned}\Pi'(e^G) &= \frac{v'(e^G)}{u'(w^G)},\\ \Pi'(e^B) &= \frac{kv'(e^B)}{u'(w^B)} + \frac{p(k-1)v'(e^B)}{(1-p)u'(w^G)}.\end{aligned}$$

- *The effort choice of the good type is always efficient*.
- The effort choice of the bad type is inefficient. It is *lower than the efficient effort level*.

Optimal Contract (Cont.)



Summary

- Asymmetric information may result in a market collapse.
- In the market for lemons, the fact that sellers have private information on the quality of the car prevents the market from functioning.
- In a signaling game, the informed agent moves first, and may reveal his type by choosing a different action for each type.
- Signaling games have two types of equilibria: separating equilibria and pooling equilibria.
- In Spence's education model (1973), the most plausible equilibrium is the separating equilibrium where only the good type gets education.

Summary (Cont.)

- A screening contract proposes two different effort-wage levels for the two different groups.
- The binding constraints are the participation constraint of the bad type and the incentive constraint of the good type.
- The bad type is pushed to the reservation utility level, whereas the good type collects an informational rent.
- The good type's effort choice is efficient, whereas the bad type's effort choice in the optimal contract is too low.